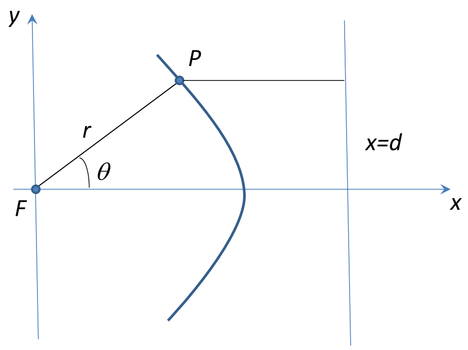
Homework #4
phy 5246
due: Friday, October 13 (in class)

P1: A particle of mass $m$ moves in a central force field given by the potential

$$V(r) = -k\frac{e^{-ar}}{r}; \quad k > 0,\ a > 0.$$

(a) Determine the effective one-dimensional problem for the radial coordinate. Sketch the effective potential for various values of the angular momentum $\ell$ and discuss qualitatively the possible orbits of the system.
(b) Determine the condition for a stable circular orbit. This condition should be expressed in terms of the radius of the orbit and the constant $a$.
(c) Find the period of small radial oscillations about these circular orbits.




P2: Let $F$ be a fixed point (the focus) and $\ell$ the fixed line (the directrix) which does not pass through $F$. Without loss of generality, you can choose them to be at the origin and at $x = d$, respectively (see the figure). Let $e$ be a positive number (the eccentricity) and consider the set of points $P$ that satisfy

$$\frac{\text{distance form } P \text{ to } F}{\text{distance form } P \text{ to } \ell} = e.$$

a) Show that the set of all points that satisfy the above, is described by the polar equation

$$r = \frac{ed}{1 + e\cos\theta}.$$

b) Show that if $0 < e < 1$, the equation is an ellipse of eccentricity $e$ by recasting the above into the equation

$$\frac{(x+c)^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1.$$

Determine $a$ and $c$.

c) Show that if $e = 1$, the equation represents a parabola given by

$$y^2 = -4\frac{d}{2}\left(x - \frac{d}{2}\right).$$

d) Show that if $e > 1$, the equation is a hyperbola of eccentricity $e$ by recasting the above into the equation

$$\frac{(x-c)^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1.$$

Determine $a$ and $c$.

From Goldstein Poole, and Safko, Classical Mechanics (Third Edition):
P3: Chapter 3 Problem 11.
P4: Chapter 3 Problem 14.